Binomial Theorem

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1 Introduction

1.1 Binomial Expression :

An algebraic expression consisting of only two term is called a binomial expression.
Example : (i) x + y (ii) 4x - 3y
(iii) x^2 + y^2 (iv) x^2 - $\frac{1}{a^2}$

1.2 Binomial Theorem :

The formula by which any power of a binomial expression can be expanded in the form of a series is known as binomial theorem.This theorem was given by Sir Issac Newton.

Statement : If n is a positive integer,

$$(a+x)^n = {}^nc_0 a^n x^0 + {}^nc_1 a^{n-1} x^1 + {}^nc_2 a^{n-2} x^2 + ... + {}^nc_n a^0 x^n \quad (1)$$

or,

$$(a+x)^n = a^n + {}^nc_1 a^{n-1} x + {}^nc_2 a^{n-2} x^2 + ... + x^n$$

[put ${}^nc_0 = 1$ and ${}^nc_n = 1$]

General term in the expression of $(a+x)^n$

$t_r = {}^nc_{r-1} a^{n-r+1} x^{r-1}$

1.3 Some special forms of Binomial Theorem :

I. Putting -x in place of x in equation (1), we get

$$(a-x)^n = {}^nc_0 a^n x^0 - {}^nc_1 a^{n-1} x^1 + {}^nc_2 a^{n-2} x^2 - .... + (-1)^n c_n a^0 x^n$$

II. Putting a = 1 in equation (1), we get

$$(1+x)^n = {}^nc_0 + {}^nc_1 x^1 + {}^nc_2 x^2 + {}^nc_3 x^3 .... + {}^nc_n x^n \quad (2)$$

III. Putting -x in place of x in (2), we get

$$(1-x)^n = {}^nc_0 - {}^nc_1 x^1 + {}^nc_2 x^2 - {}^nc_3 x^3 .... + (-1)^n c_n x^n \quad (3)$$

IV. Putting the values of ${}^nc_0, {}^nc_1, {}^nc_2, ...{}^nc_n$
Now, ${}^nc_0$

$$ {}^nc_0 = \frac{n!}{0!(n-0)!} $$
$$ {}^nc_0 = 1 $$

For, ${}^nc_1$

$$ {}^nc_1 = \frac{n!}{1!(n-1)!} $$
$$ or,\ \ {}^nc_1 = \frac{n(n-1)!}{1!(n-1)!} $$
$$ {}^nc_1 = n $$

For, ${}^nc_2$

$$ {}^nc_2 = \frac{n!}{2!(n-2)!} $$
$$ or,\ \ {}^nc_2 = \frac{n(n-1)(n-2)!}{2!(n-2)!} $$
$$ {}^nc_2 = \frac{n(n-1)}{2!} $$

Then, we have

$$ (x+a)^n = x^n + nx^{n-1}a + \frac{n(n-1)}{2!}x^{n-2}a^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}a^3 + ... + a^n $$